Diagonalization and related Problems

$$\begin{bmatrix} x_{1a} & x_{2a} & x_{3a} \\ x_{1b} & x_{2b} & x_{3b} \\ x_{1c} & x_{2c} & x_{3c} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} x_{1a}\lambda_1 & x_{2a}\lambda_2 \\ x_{1b}\lambda_1 & x_{2b}\lambda_2 & \cdots \\ x_{1c}\lambda_1 & x_{2c}\lambda_2 \end{bmatrix}$$

$$\boxed{\begin{gathered} AX = X\Lambda \\ A = X\Lambda X^{-1} \end{gathered}}$$

$$A^2 = A \cdot A = (X\Lambda \underbrace{\cancel{X^{-1}})(\cancel{X}}_{I} \Lambda X^{-1})$$

$$A^2 = X\Lambda^2 X^{-1}$$

$$A^3 = A^2 \cdot A = (X\Lambda^2 \cancel{X^{-1}})(\cancel{X}\Lambda X^{-1})$$

$$= X\Lambda^3 X^{-1}$$

$$\boxed{A^n = X\Lambda^n X^{-1}}$$

$$\Lambda^n = ? = \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \cdots \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{n}$$

$$= \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix} \cdots \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{bmatrix}$$

$\Lambda^n = \underline{\text{L.}\wedge\text{n}}$ in MATLAB

What if A is symmetric?

$A^T = A \Rightarrow X$ call Q

$Q^{-1} = Q^T$

$$\boxed{\begin{aligned} A &= Q \Lambda Q^T \\ A^n &= Q \Lambda^n Q^T \end{aligned}}$$

No Repeating eigenvalues

$\Rightarrow X$ is singular, no X^{-1}

Change of Basis

$AX = X\Lambda$

$Av = A\underline{Xc} \leftarrow v = Xc$

$Av = X\Lambda c$

$A^2 v = AX\Lambda c = X\Lambda\Lambda c = X\Lambda^2 c$

$A^k v = X \Lambda^k c$