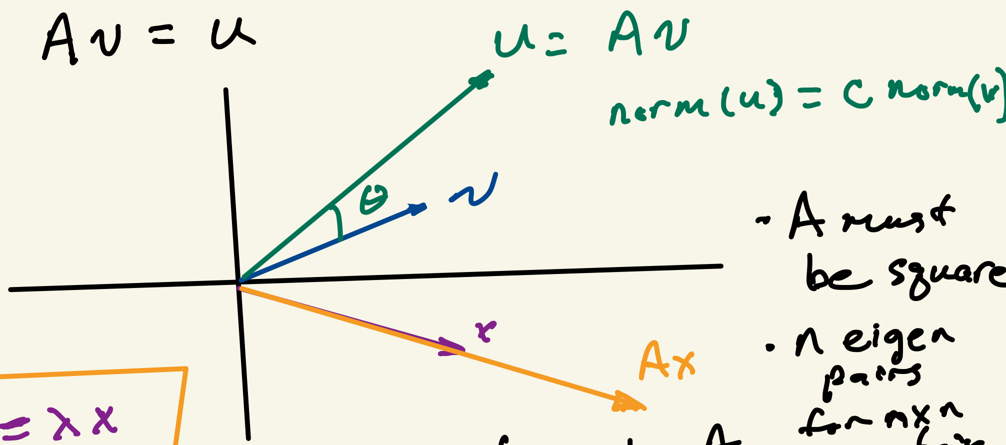
Eigenvalues & Eigenvectors

$Av = u$



- A must be square
- n eigen pairs for n x n matrix

$\boxed{Ax = \lambda x}$

x - eigenvectors to A

λ - eigenvalues of A

$\cancel{A = \lambda}$

when Ax, replace A w/ λ → λx

Eigenvectors invariet to Scaling

$$Ax = \lambda X \Rightarrow AcX = \lambda c X$$

Finding Eigenvalues

$$Ax = \lambda x \qquad Ax - \lambda I x = 0$$

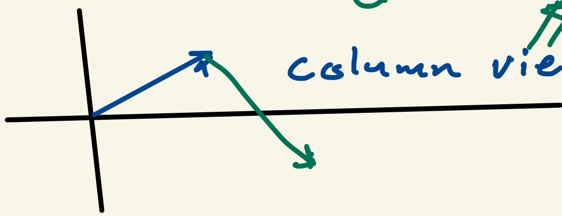
$$\begin{bmatrix} \lambda x_1 \\ \lambda x_2 \\ \lambda x_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} \lambda & & 0 \\ & \lambda & \\ 0 & & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\underbrace{(A - \lambda I)}_{C}x = 0$$

C - characteristic matrix

1. ~~$C = 0$~~
2. ~~$x = 0$~~
3. linear comb $Cx = 0$

C must be singular



column view of matrix, vector multiplication

When is C singular?

$(A - \lambda I)$

1. rank(A) < size(A)

no algorithm to find λ.

2. det(C) = 0

   ok for 2x2!

3. QR algoritm.

$$C = \begin{bmatrix} 2-\lambda & 1 \\ 3 & -1-\lambda \end{bmatrix}$$

$$|C| = (2-\lambda)(-1-\lambda) - 3 = 0$$

$$= -2 - 2\lambda + \lambda + \lambda^2 - 3$$

$$|C| = \lambda^2 - \lambda - 5 = 0$$

$$\lambda = \frac{1 \pm \sqrt{1+20}}{2} \simeq 2.7913, -1.793$$