LU Decomposition

$Ax = b$

Wrong - slow

$x = A^{-1}b$

right in MATLAB

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x = A\b;
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[A b] - elimination

$$\begin{bmatrix} \ddots & \vdots \\ 0 & \ddots \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} c \end{bmatrix}$$

$Ux = c$

[A] - elimination

$EA = U$

E elimination matrix

$A = E^{-1}U = LU$

$$L = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} \qquad U = \begin{bmatrix} \ddots & \vdots \\ 0 & \ddots \end{bmatrix}$$

L lower Δ

U

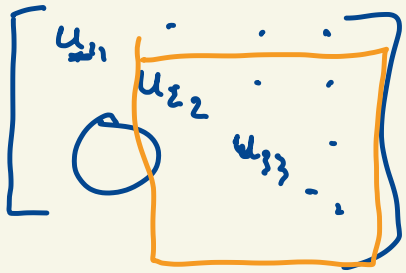
$LUx = b$

$x = L^{-1}U^{-1}b$

$x = L\backslash(U\backslash b)$

Applications

1. fixed A, lots of b - $Ax = b$
2. Determinant shortcut

$U =$ 

$\det(U)$?

u_{11} $\begin{bmatrix} u_{22} & \cdots \\ \vdots & \ddots \end{bmatrix}$

$\det(U) = \prod(\text{diag}(U))$

$\text{prod}(\text{diag}(U))$

u_{22}

u_{33} $\begin{bmatrix} u_{44} & ? \\ 0 & u_{55} \end{bmatrix}$

$\det(L)$? $\left| \begin{bmatrix} 1 & & & 0 \\ \ddots & 1 & & \\ & \ddots & 1 & \\ & & \ddots & 1 \end{bmatrix} \right| = 1$