

Section 6.10.8

Principal Component Analysis

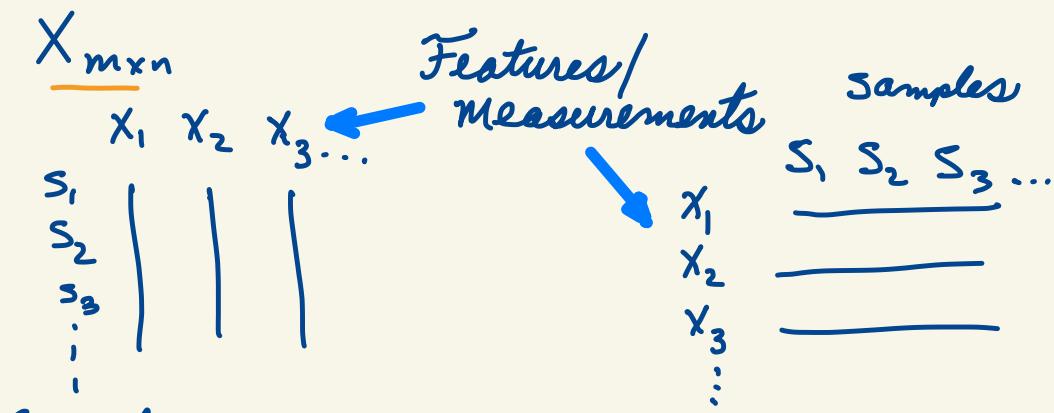
PCA

PCA gives us:

- Dimensionality Reduction
- PCA Space
- What things are similar and different?

Applications:

- Identification
- Classification
- Recommender Service



* zero mean for each feature
repmat \rightarrow matrix of means

$$D = \begin{bmatrix} & & \\ | & | & | \\ (x_1 - \mu_1) & (x_2 - \mu_2) & \dots \\ | & | & | \\ & & \end{bmatrix} \quad D = \begin{bmatrix} -(x_1 - \mu_1) \\ -(x_2 - \mu_2) \\ \vdots \end{bmatrix}$$

$$\text{cov}_X = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_2 x_1} & \sigma_{x_3 x_1} \dots \\ \sigma_{x_1 x_2} & \sigma_{x_2}^2 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$\text{cov}_X = \frac{D^T D}{m-1}$$

$$\text{cov}_X = \frac{DD^T}{n-1}$$

- * $[V, L] = \text{eig}(\text{cov}_X);$
- * sort V by eigenvalues
- * limit W to top k columns of V based on the eigenvalues

$$W = [v_1 \ v_2 \ \dots \ v_k]$$

$$Y = DW$$

$$Y = W^T D$$

— The PCA Space —

- Y has smaller dimension than X .
- Y is a linear combination of D data.
- Y shows the Principal component of the data.

To reconstruct \tilde{x} in reduced dimension: usually
not
needed

* Reverse the steps

$$\tilde{Y} = D + \mu$$

$$Y = DW$$

$$D = YW^T$$

$$\hat{Y} = YW^T + \mu$$

$$\hat{x} = D + \mu$$

$$Y = W^TD$$

$$D = WY$$

$$\hat{x} = WY + \mu$$