8.7 The Flight of a Home Run Baseball

It was the bottom of the ninth inning of game 1 of the 2015 World Series. The New York Mets led the Kansas City Royals 4 to 3 with one out in the bottom of the ninth inning and then this happened. Watch the video and make note of two things – how long was the ball in the air (measure with a stop-watch), and estimate how far the ball went before hitting the ground. The marker on the wall gives us a starting point to estimate how far the ball went.

You are to write a MATLAB script that will plot the vertical and horizontal path of the ball. The script should also display the maximum height, initial velocity, and initial projection angle of the ball. The steps listed below will guide you.

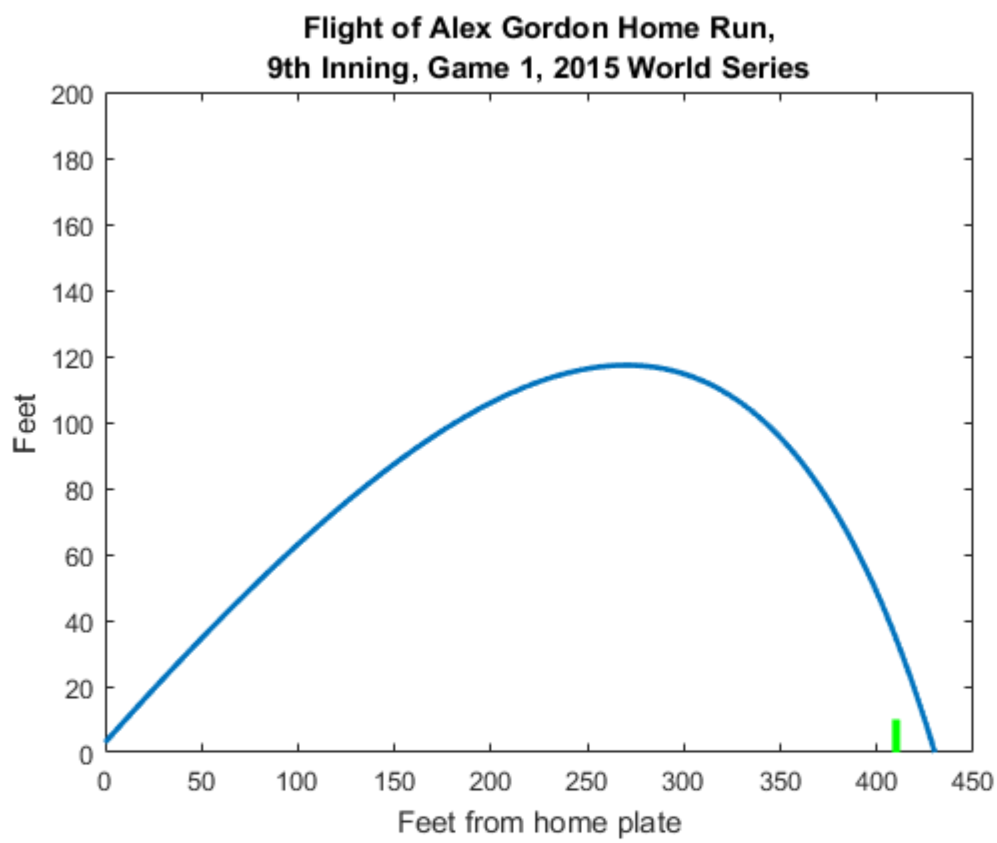

Fig. 8.1: Example Plot

The horizontal and vertical paths of the ball may be calculated independently.

1. First make variables for constant values – gravity: $g = -32 feet/sec^2$, final time of flight: t_f in seconds, Initial height: $y_0 = 3 feet$, and the final horizontal displacement: x_f in feet. Also use `linspace` to make a 100 element vector, t for time.
2. The vertical path of the ball may be treated the same as for throwing a ball straight up. The

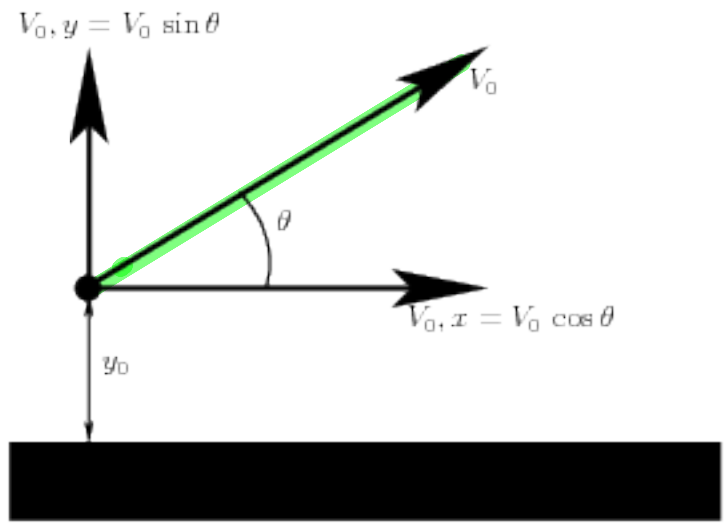

Fig. 8.2: The ball begins at an initial height. It has an initial velocity at an initial projection angle. The initial velocity may be split into horizontal and vertical components.

$y(t_f) = 0$ ① $(-y_0 - 1/2\, g t_f^2)/t_f = V_{0,y}$

only force on the ball worth noting is gravity. The height of the ball is given by

$$② \quad y = y_0 + V_{0,y}\, t + 1/2\, g\, t^2$$

Using the t_f value measured with the stop watch, calculate $V_{0,y}$, which occurs when $y = 0$. The y path of the ball may be calculated now.

3. Next use the final horizontal displacement, x_f, to calculate the initial horizontal velocity. Air resistance will slightly slow the horizontal displacement *[ADAIR90]* according to the differential equation,

   $x(t_f) = x_f$

   $$a = \frac{dv_x(t)}{dt} = -0.2\, v_x(t).$$

   The solution to the differential equation is

   $$v_x(t) = V_{0,x}\, e^{-0.2\, t}.$$

   ③ $V_{0x} = \dfrac{x_f}{5(1 - e^{-0.2 t_f})}$

   Then, the displacement comes from the definite integral of the velocity.

   $$④ \quad x(t) = V_{0,x} \int_0^t e^{-0.2\tau} d\tau = 5\, V_{0,x} (1 - e^{-0.2\, t}).$$

   From simple trigonometry, V_0 can be expressed in terms of $V_{0,y}$, $V_{0,x}$, and θ. Then expand the algebra to find an equation for θ using $v_{0,y}$ and $v_{0,x}$. Hint: Use the `atan2d` function.

4. Plot the path with a `'LineWidth'` of 2.

   Show the home run wall by plotting a green line with `'LineWidth'` of 3. Make the wall 10 feet tall at 410 feet from home plate.

   Remember to annotate the plot and to report the maximum height, initial velocity, and initial projection angle of the ball.