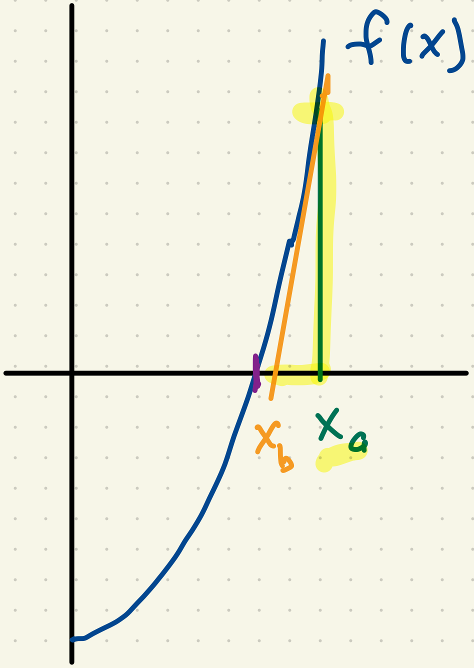

Newton's algorithm to find a root of a function.

$$\text{Slope} = f'(x_a) = \frac{\Delta y}{\Delta x}$$

$$f'(x_a) = \frac{f(x_a)}{x_a - x_b}$$

$$x_b = x_a - \frac{f(x_a)}{f'(x_a)}$$

$$x_a = x_b$$

loop until $f(x_a) \approx 0$

$$x = \sqrt{K}$$

$$f(x) = x^2 - K$$

$$f'(x) = 2x$$

$$x_b = x_a - \frac{x_a^2 - K}{2x_a}$$

$$x_b = \frac{2x_a^2 - x_a^2 + K}{2x_a}$$

$$x_b = \frac{x_a^2 + K}{2x_a} = \boxed{\frac{x_a + \frac{K}{x_a}}{2}}$$

when $x_a = \sqrt{K}$

$$x_b = \frac{\sqrt{K} + \frac{K}{\sqrt{K}}}{2} = \sqrt{K}$$

$x \stackrel{?}{=} \sqrt{9}$, $K = 9$

$$X_a = \frac{K}{2} = \frac{9}{2} = 4.5$$

$$X_b = \frac{4.5 + \frac{9}{4.5}}{2} = 3.25$$

$$X_a = 3.25$$

$$X_b = \frac{3.25 + \frac{9}{3.25}}{2} = 3.0096$$

$$X_a = 3.0096 \qquad \sqrt{9} = 3$$

What about $\sqrt{K}$ is irrational ($\sqrt{2}, \sqrt{5}, \sqrt{7}, \ldots$)

$$X_a \approx \sqrt{K}$$

$$f(X_a) = \left| \frac{X_a^2 - K}{K} \right| < \epsilon$$

ϵ: small number